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Extension of a Method for Determination of Flight Equipment Acceleration

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Nomenclature

$\ddot{a}(\omega)$	=	input acceleration
C	=	number of components
$\{F_r\}$	=	reaction forces
$\{f\}$	=	excitation for the coupled structure
f_k, ω_k	=	hard-mounted natural frequency of kth component
$[K]$	=	stiffness matrix of secondary structure
k_k	=	stiffness of the connection between kth component and secondary structure
L_j	=	jth modal participation factor of coupled structure
$[l]$	=	modal participation factors of secondary structure
$[M]$	=	secondary structure mass matrix
$[M_c]$	=	mass matrix related to the components
$[\bar{M}_c]$	=	$[M_c]$ partitioned
m_k	=	mass of kth component
N	=	number of modes of the secondary structure
$\{q\}$	=	generalized coordinates of elastic modes
$\{q_r\}$	=	generalized coordinates of grounded nodes
r_j	=	generalized coordinate of the coupled structure
$S(\omega)$	=	input power spectral density
$[V_c]$	=	transformed mass matrix of components
$[\bar{V}_c]$	=	$[V_c]$ partitioned
$\{y\}$	=	degrees of freedom (DOF) of the secondary structure
$\{y_c\}$	=	displacements at the components' connection points
$\{y_i\}$	=	internal DOF of the secondary structure
$\{y_r\}$	=	grounded DOF of the secondary structure
$\{\delta\}$	=	components' relative displacements, $\{\delta_a\} - \{y_c\}$
$\{\delta_a\}$	=	components' absolute displacements
ζ	=	viscous damping ratio
$[\Lambda]$	=	eigenvalue matrix (diagonal) of the grounded secondary structure
σ_{δ_k}	=	standard deviation of the absolute acceleration of kth component
$[\Phi]$	=	mass normalized mode shapes of the secondary structure
$[\Phi_{rr}]$, $[\Phi_{ir}]$	=	translational matrices
$[\Psi]$	=	mass normalized mode shapes of the coupled structure
ω_j^2	=	jth eigenvalue of the coupled structure

Subscripts and Superscripts

c	=	component
i	=	ith internal DOF of the secondary structure
j	=	jth eigenmode of the coupled system
k	=	kth component
r	=	rth grounded DOF of the secondary structure

Introduction

DURING the launch, loads from different sources excite spacecraft primary and secondary structures. In particular, random oscillations of the primary structure, assumed known during the preliminary design, can be considered as a random vibration environment for secondary structures, for example, mounting frames. This excitation is of great importance for the determination of load factors acting on flight equipment, for example, electronic boxes, batteries, and scientific instruments, that are supported by secondary structures.

As described in Ref. 1, several techniques are used in engineering practice to evaluate the dynamic response of components, even though it has been demonstrated that some methods are too much conservative (in particular the technique based on the use of Miles's equation[2]). To obtain an accurate estimate of the acceleration of flight equipment, recently a new, highly cost-efficient technique has been proposed by Ruotolo and Cotterchio.[3] It has the main advantage of requiring only a few properties of the secondary structure to be grounded at the interface with the primary structure, so that it permits optimization runs aimed at mass saving.

Nevertheless, the formulation of the method described in Ref. 3 permits consideration of only a single piece of flight equipment connected to the secondary structure, so that its use is limited in practical applications. As a consequence, in this Note the technique is extended to deal with several components, permitting the consideration of real space structures, for example, nodes 2 and 3 of the International Space Station.

Determination of Components' Acceleration

Figure 1 shows a number of components connected to a secondary structure. It is assumed that stiffness and mass matrices of the latter structure are known so that its potential and kinetic energy can be written as

$$U_s = \tfrac{1}{2}\{y\}^T[K]\{y\}, \qquad T_s = \tfrac{1}{2}\{\dot{y}\}^T[M]\{\dot{y}\} \tag{1}$$

In the following discussion, it is assumed that the kth component is connected to only one degree of freedom (DOF) of the secondary structure through a spring with stiffness k_k. When all DOF of the connection points are collected into the vector $\{y_c\}$ and the relative displacement between components and corresponding mounting points on the secondary structure are introduced, potential and kinetic energies related to flight equipment are given by

$$U_c = \tfrac{1}{2}\{\delta\}^T[k_c]\{\delta\}, \qquad T_c = \tfrac{1}{2}(\{\dot{y}_c\} + \{\dot{\delta}\})^T[m_c](\{\dot{y}_c\} + \{\dot{\delta}\}) \tag{2}$$

where matrices $[k_c]$ and $[m_c]$ have rigidities k_k and masses m_k on their principal diagonal. When the corresponding energies

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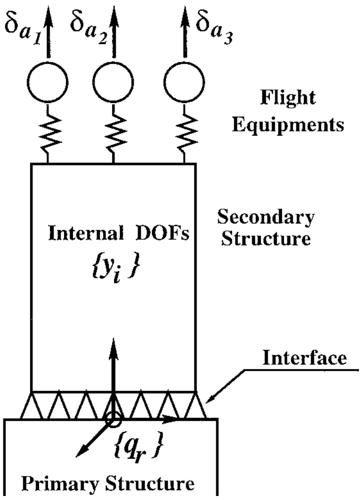

Fig. 1 Translational motion transmitted to the secondary structure.

are summed, a transformation matrix $[T]$ is introduced so that $\{y_c\} = [T]\{y\}$, and Lagrange's equations are applied, stiffness and mass matrices of the coupled structure can be obtained:

$$[K_T] = \begin{bmatrix} [K] & [0] \\ [0]^T & [k_c] \end{bmatrix}, \qquad [M_T] = \begin{bmatrix} [M] + [M_c] & [V_c] \\ [V_c]^T & [m_c] \end{bmatrix} \quad (3)$$

where $[M_c] = [T]^T[m_c][T]$ and $[V_c] = [T]^T[m_c]$.

When the same procedure as described in Ref. 3 is followed, it is assumed that every component is excited only through its interface to the secondary structure. As a result, when the DOF of the secondary structure are split into internal and grounded DOF, the equation of motion of the coupled system becomes

$$\begin{bmatrix} [K_{rr}] & [K_{ri}] & [0] \\ [K_{ir}] & [K_{ii}] & [0] \\ [0]^T & [0]^T & [k_c] \end{bmatrix} \begin{Bmatrix} \{y_r\} \\ \{y_i\} \\ \{\delta\} \end{Bmatrix} + \begin{bmatrix} [M_{rr}] & [M_{ri}] & [0] \\ [M_{ir}] & [M_{ii}] + [\bar{M}_c] & [\bar{V}_c] \\ [0]^T & [\bar{V}_c]^T & [m_c] \end{bmatrix} \begin{Bmatrix} \{\ddot{y}_r\} \\ \{\ddot{y}_i\} \\ \{\ddot{\delta}\} \end{Bmatrix} = \begin{Bmatrix} \{F_r\} \\ \{0\} \\ \{0\} \end{Bmatrix} \quad (4)$$

According to Ref. 3, it is assumed that the interface to the primary structure is either isostatic or infinitely rigid so that the interface motion can be described by only three DOF, as shown in Fig. 1: translations along x, y, and z directions are collected into $\{q_r\}$ (corresponding rotations are neglected). As a consequence, the movement of the secondary structure is given by the superposition of the rigid-body motion due to the interface to the primary structure and the elastic movement with respect to the interface. The last motion is expressed by taking advantage of the elastic modes of the secondary structure fixed at the interface. As a result, the following transformation of variables can be applied:

$$\begin{Bmatrix} \{y_r\} \\ \{y_i\} \\ \{\delta\} \end{Bmatrix} = \begin{bmatrix} [\Phi_{rr}] & [0] & [0] \\ [\Phi_{ir}] & [\Phi] & [0] \\ [0]^T & [0]^T & [I] \end{bmatrix} \begin{Bmatrix} \{q_r\} \\ \{q\} \\ \{\delta\} \end{Bmatrix} = [\mathcal{T}] \cdot \begin{Bmatrix} \{q_r\} \\ \{q\} \\ \{\delta\} \end{Bmatrix} \quad (5)$$

where $[\Phi_{rr}]$ and $[\Phi_{ir}]$ are matrices collecting zero and unity elements according to the DOF orientation, $[\Phi]$ is the matrix containing the mass normalized mode shapes of the secondary structure where all of the DOFs of the interface are eliminated.

To determine the dynamic response of components and of the secondary structure, Eq. (5) is introduced into Eq. (4), and the latter is multiplied by $[\mathcal{T}]^T$. The second and third equation of the consequent system of equations are

$$\begin{bmatrix} [\Lambda] & [0] \\ [0]^T & [k_c] \end{bmatrix} \begin{Bmatrix} \{q\} \\ \{\delta\} \end{Bmatrix} + \begin{bmatrix} [I] + [\Phi]^T[\bar{M}_c][\Phi] & [\Phi]^T[\bar{V}_c] \\ [\bar{V}_c]^T[\Phi] & [m_c] \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\} \\ \{\ddot{\delta}\} \end{Bmatrix} = -\begin{bmatrix} [l] + [\Phi]^T[\bar{M}_c][\Phi_{ir}] \\ [\bar{V}_c]^T[\Phi_{ir}] \end{bmatrix} \{\ddot{q}_r\} \quad (6)$$

where the modal participation factors matrix, related to base excitation of only the secondary structure, is defined as

$$[l] = [\Phi]^T([M_{ii}][\Phi_{ir}] + [M_{ir}][\Phi_{rr}])$$

To determine the dynamic response of the whole structure, forced equation (6) is solved by using modal superposition. First, the homogeneous system of equations in Eq. (6) is solved to determine the eigenvalues and corresponding mass-normalized eigenvectors of the complete structure. Second, the generalized coordinates of the complete structure, given by

$$[\Psi]\{r\} = \begin{Bmatrix} \{q\} \\ \{\delta\} \end{Bmatrix}$$

are introduced, where every generalized coordinate r_j is

$$r_j = \frac{\{\Psi_j\}^T\{f\}}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j}$$

$$\{f\} = -\begin{bmatrix} [l] + [\Phi]^T[\bar{M}_c][\Phi_{ir}] \\ [\bar{V}_c]^T[\Phi_{ir}] \end{bmatrix} \{\ddot{q}_r\} \quad (7)$$

and a viscous damping ratio has been introduced to represent as accurately as possible the dynamic response of real structures.

Usually the excitation to the secondary structure is defined in terms of a single shape in frequency, so that the structure motion along x, y, and z directions can be rewritten as

$$\{\ddot{q}_r\} = \lfloor R_x \quad R_y \quad R_z \rfloor^T \ddot{a}(\omega) \quad (8)$$

As a consequence, modal participation factors of the coupled structure are

$$L_j\ddot{a} = \{\Psi_j\}^T\{f\} \quad (9)$$

When it is assumed that C components are connected to the secondary structure, the relative displacement for the kth flight equipment can be evaluated by summing the contribution of every mode of the homogeneous equation (6):

$$\delta_k(\omega) = \sum_{j=1}^{N+C} \Psi_{N+k,j} \cdot r_j = \sum_{j=1}^{N+C} \left(\frac{\Psi_{N+k,j} L_j}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j} \right) \ddot{a}(\omega) \quad (10)$$

where it has been assumed that the secondary structure is represented through N principal modes and that $\Psi_{N+k,j}$ is the element of the jth mode shape of the complete structure in correspondence of the kth component. As a result, the corresponding absolute acceleration is given by[3]

$$\ddot{\delta}_{a,k}(\omega) = \sum_{j=1}^{N+C} \left[-\Psi_{N+k,j} L_j \frac{\omega_k^2 + 2i\zeta\omega\omega_k}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j} \right] \ddot{a}(\omega) \quad (11)$$

From the last equation, the power spectral density of the acceleration of the kth component can be derived, which permits evaluation of the variance of the absolute acceleration due to a random process with power spectral density given by $S(\omega)$ and exciting the secondary structure:

$$\sigma_{\ddot{\delta}_k^2} = 2\int_0^\infty \left| \sum_{j=1}^{N+C} \left(-\Psi_{N+k,j} L_j \frac{\omega_k^2 + 2i\zeta\omega\omega_k}{-\omega^2 + \omega_j^2 + 2i\zeta\omega\omega_j} \right) \right|^2 S(\omega)\,d\omega \quad (12)$$

Note that, as in the case of a single supported component, effective masses[4,5] of the secondary structure, given by $[l]^T[l]$, are of great importance in the selection of the N modes to be included during the evaluation of the flight equipment acceleration because they are present in the L_j factor. In particular, given a certain direction for the motion of the base, only modes with effective masses greater than about 0.5% should be considered for the analysis.

Numerical Examples

To illustrate results obtained by using this procedure, the same truss analyzed in Ref. 3 with mass $M = 71.5$ kg has been used; it is shown in Fig. 2, and the main properties of its first modes are listed in Table 1. It is assumed that the structure is excited along the x direction by a random process with power spectral density $S(f)$ equal to 0.02 g^2/Hz in the range from 1 to 500 Hz. Two pieces of flight equipment with mass $m_1 = m_2 = 0.71$ kg have been connected to this secondary structure, at nodes 8 and 11, respectively, and it is assumed that both are directed along the x direction. As a result, in accord with Table 1 and when only those modes with effective mass greater than 0.5% for an excitation along the x direction are considered, only $N = 6$ modes are used to represent the dynamic properties of the truss.

The standard deviation of the absolute acceleration of component 2 (located at node 11) is shown in Fig. 3 as a function of hard-mounted natural frequencies, f_1 and f_2, of both pieces of flight equipment. Figure 3 highlights the dynamic coupling effect due to the presence of two components; in particular, it is possible to observe that the condition $f_1 = f_2$ permits to reduce the acceleration of components with respect to the case $f_1 \neq f_2$. Moreover, these results show that the maximum dynamic response of component 2 can be obtained with $f_2 \approx 140$ Hz, when $f_2 > f_1$, that is, a weak connection for component 1, and with $f_2 \approx 120$ Hz, when $f_2 < f_1$, that is, a stiff connection for component 1. When the data listed in Table 1 are considered, it follows that, when several components are present, their maximum dynamic response may be obtained for hard-mounted natural frequencies different from the natural frequency of the principal modes as a consequence of dynamic coupling effects.

Table 1 Natural frequencies and corresponding effective masses for the secondary structure under analysis

Mode no.	Natural frequency, Hz	Effective mass, direction X	Effective mass, direction Y
1	22.5	0.08	64.61
2	91.9	2.73	20.16
3	144.4	77.39	0.63
4	191.6	0.02	4.95
5	277.3	0.12	1.94
6	346.6	0.51	0.52
7	391.7	2.15	0.43
8	406.2	3.61	0.01
9	458.1	2.46	0.04

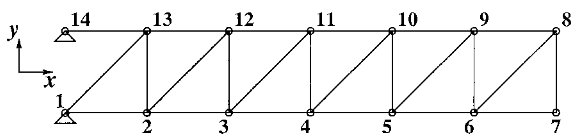

Fig. 2 Secondary structure under analysis.

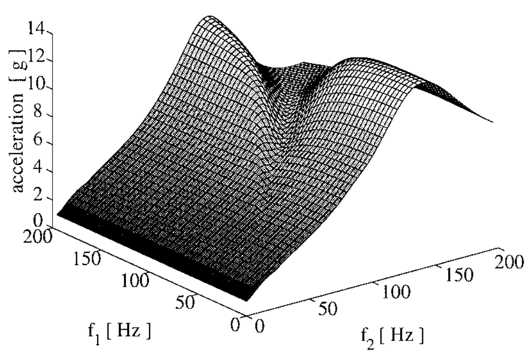

Fig. 3 Acceleration of component 2.

Conclusions

In this Note, an extension of a previously documented procedure aimed at the evaluation of the dynamic response of flight equipment excited by random vibration has been presented. This extended technique maintains the same main features of the previous method: the high computational efficiency with respect to a classical approach based on the use of a finite element code, the usefulness of effective mass information, and the possibility of exciting the structure along one direction and locating a component along a different direction. Furthermore, this simple improvement is of great importance from a practical point of view because real engineering applications can be dealt with by considering dynamic coupling effects due to the presence of several components. Eventually, these properties open new possibilities in terms of optimizing the location of a number of components and of addressing sensitivity problems of their dynamic response with respect to the stiffness of the connections.

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